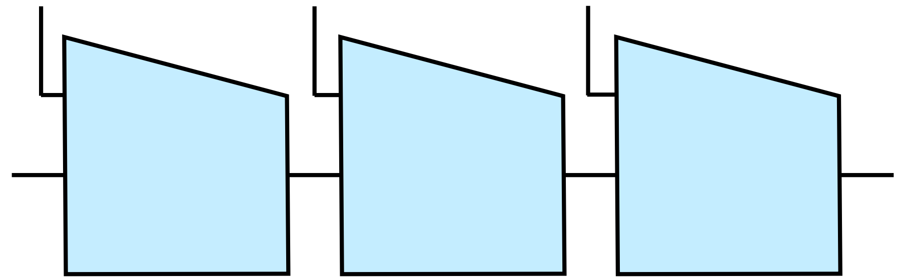
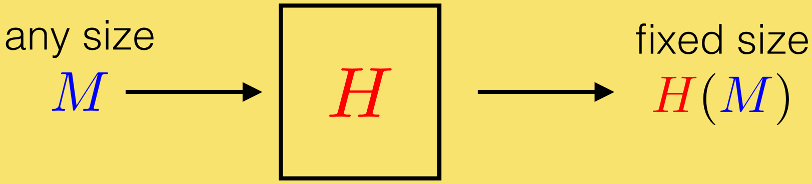


Better Than Advertised:
Improved Collision-Resistance Guarantees for MD-Based Hash Functions

Mihir Bellare   Joseph Jaeger   **Julia Len**

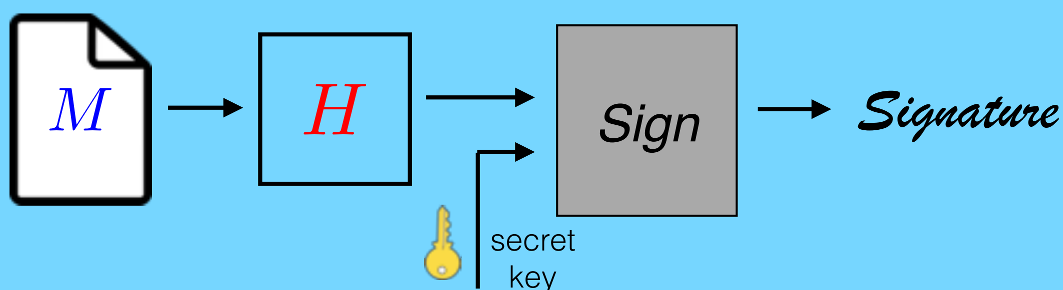
UC San Diego

Hash Functions



$$H : \{0,1\}^* \to \{0,1\}^n$$

Central usage: Certificates



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Hard to find distinct messages with the same hash in time less than $2^{n/2}$, the time of a birthday attack.

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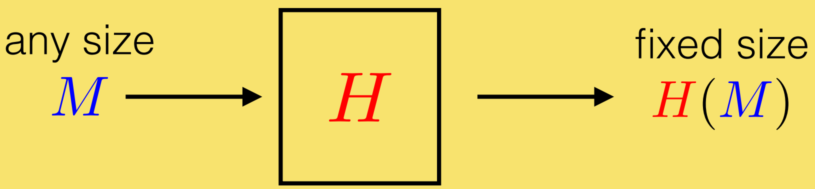

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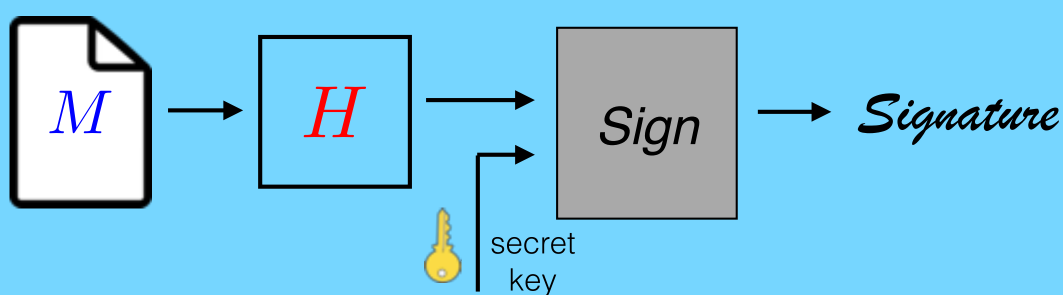

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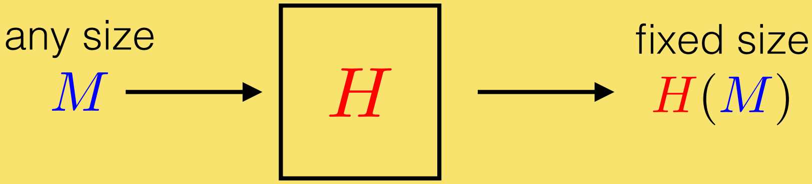

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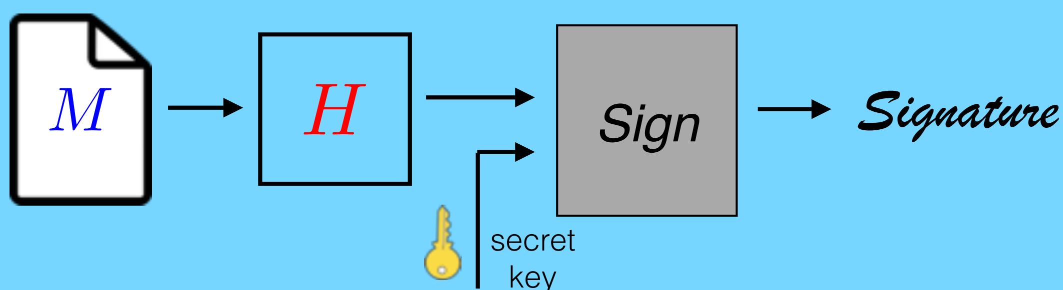

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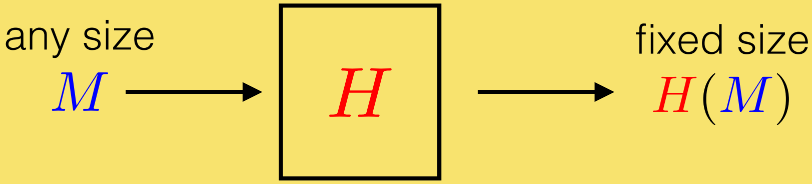

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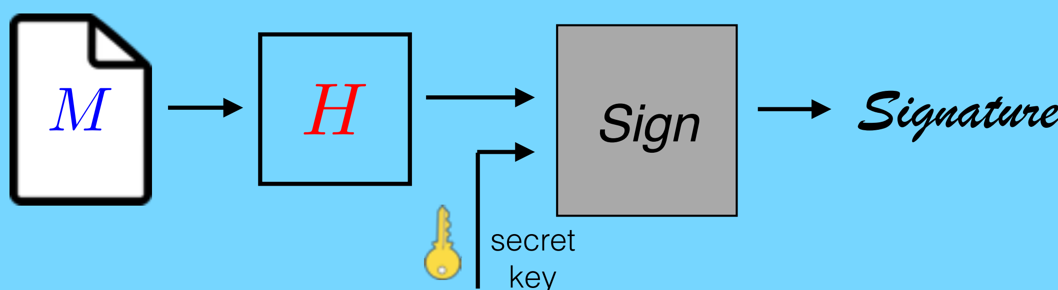

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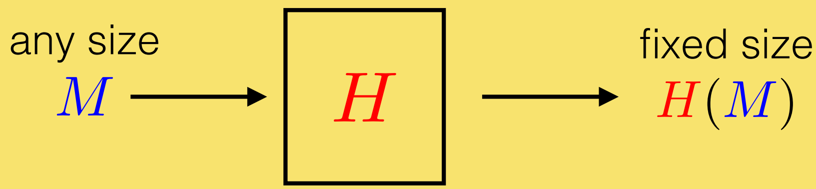

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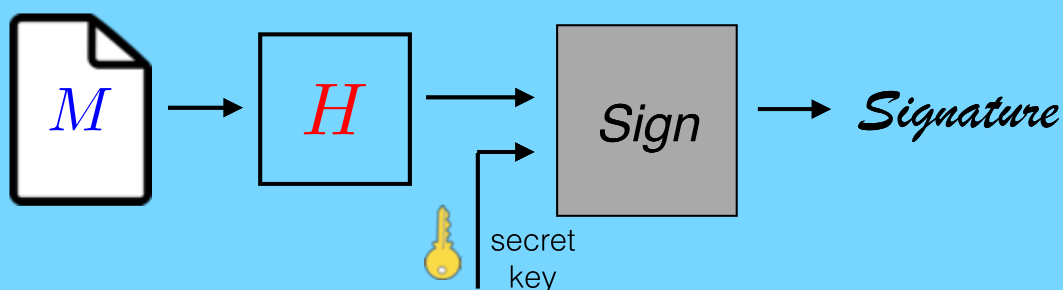

Collisions in H lead to certificate forgery. SHA-1 collision leading to browsers no longer accepting SHA-1-based certificates.

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[SBKAM17]

https://shattered.io/

How hash functions are built

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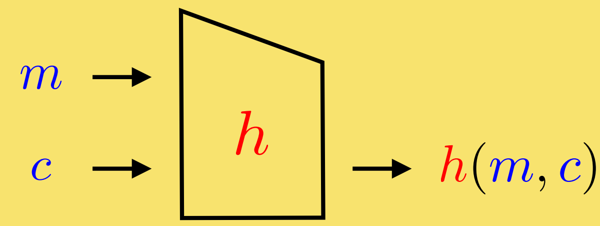

$$h : \{0,1\}^{h.ml + h.cl} \rightarrow \{0,1\}^{h.cl}$$

H	$h.ml$	$h.cl$
MD5	512	128
SHA-1	512	160
SHA-256	512	256
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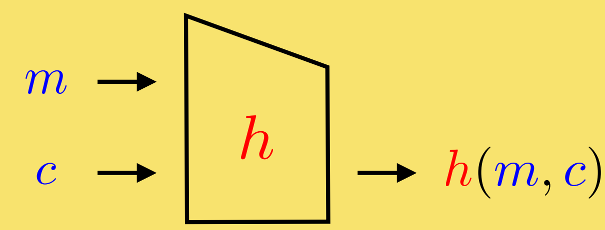

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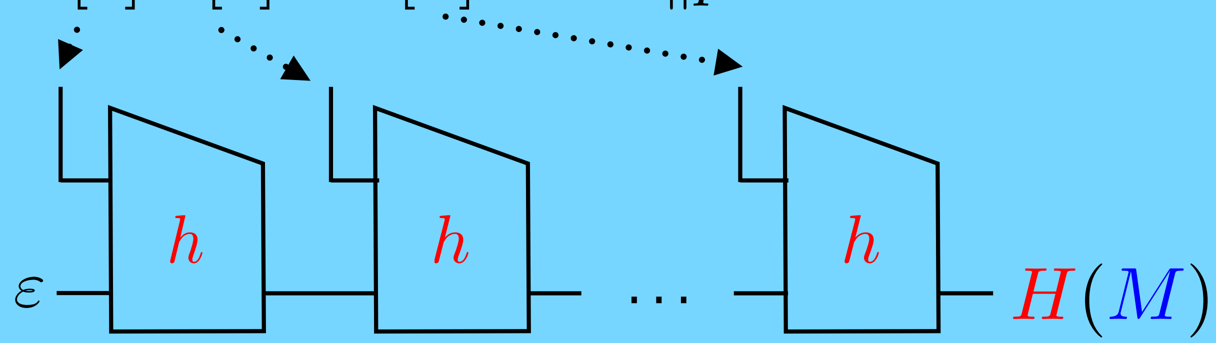

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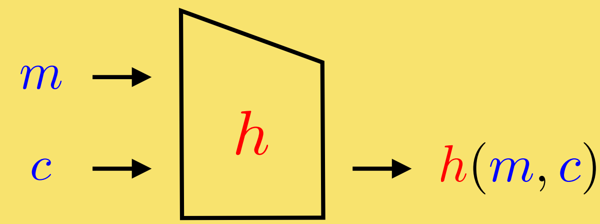

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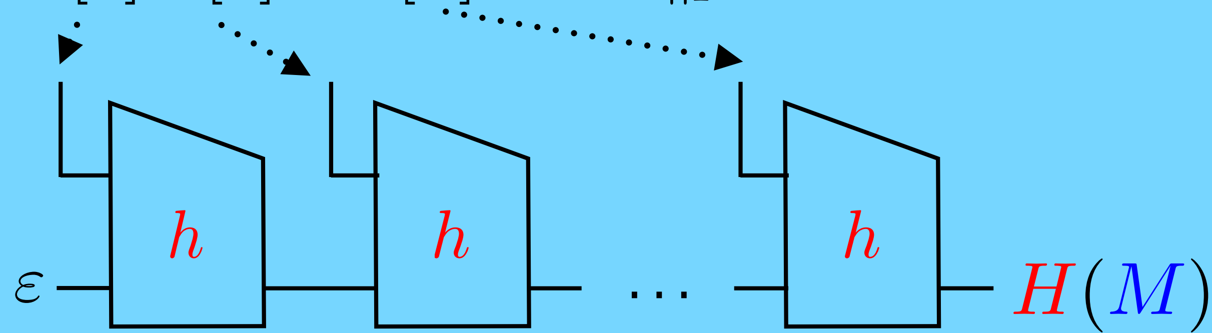


Merkle


Damgård

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Our Answer: YES, X = CCR

Constrained **C**ollision-**R**esistance.
We will define this and show it is weaker than CR.

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Security amplification: The MD transform "amplifies" or "boosts" security by turning a weaker-than-CR compression functions into a CR hash function.

Contributions

Our Theorem 1: h CCR => H CR

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These results are obtained via a **general framework**

— Parameterized version of MD: $H = \mathbf{MD}[h, \mathsf{Split}, \mathsf{S}]$

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Some of our **other results**

— We give an MD variant that is more efficient than MD

— Memory-efficient reductions

— Various separations and counter-examples

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5. A lot of our work formalizes, extends and unifies folklore or known results.
Nothing we do is technically hard.

The MD Framework

Splitting function $\mathsf{Split} : D \rightarrow (\{0,1\}^{h.ml})^*$

Set of starting points $\mathsf{S} \subseteq \{0,1\}^{h.cl}$

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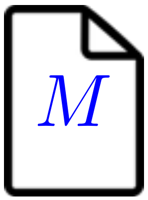

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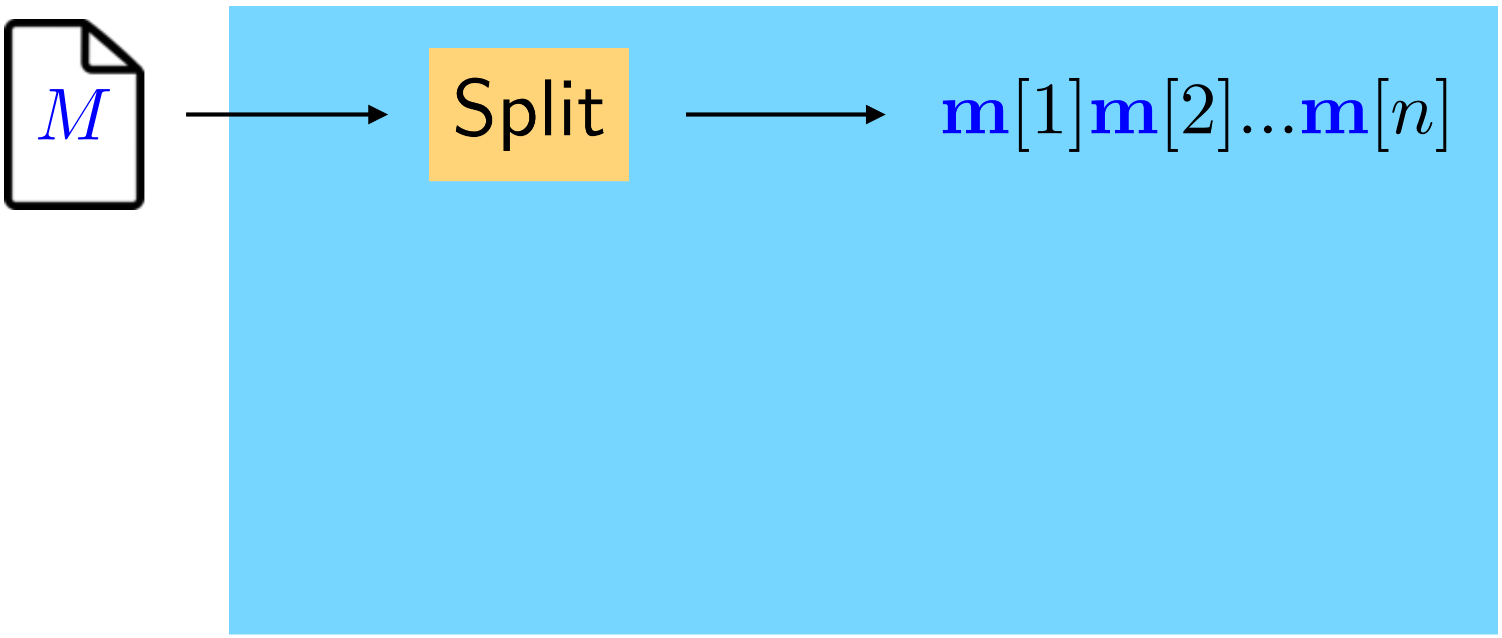

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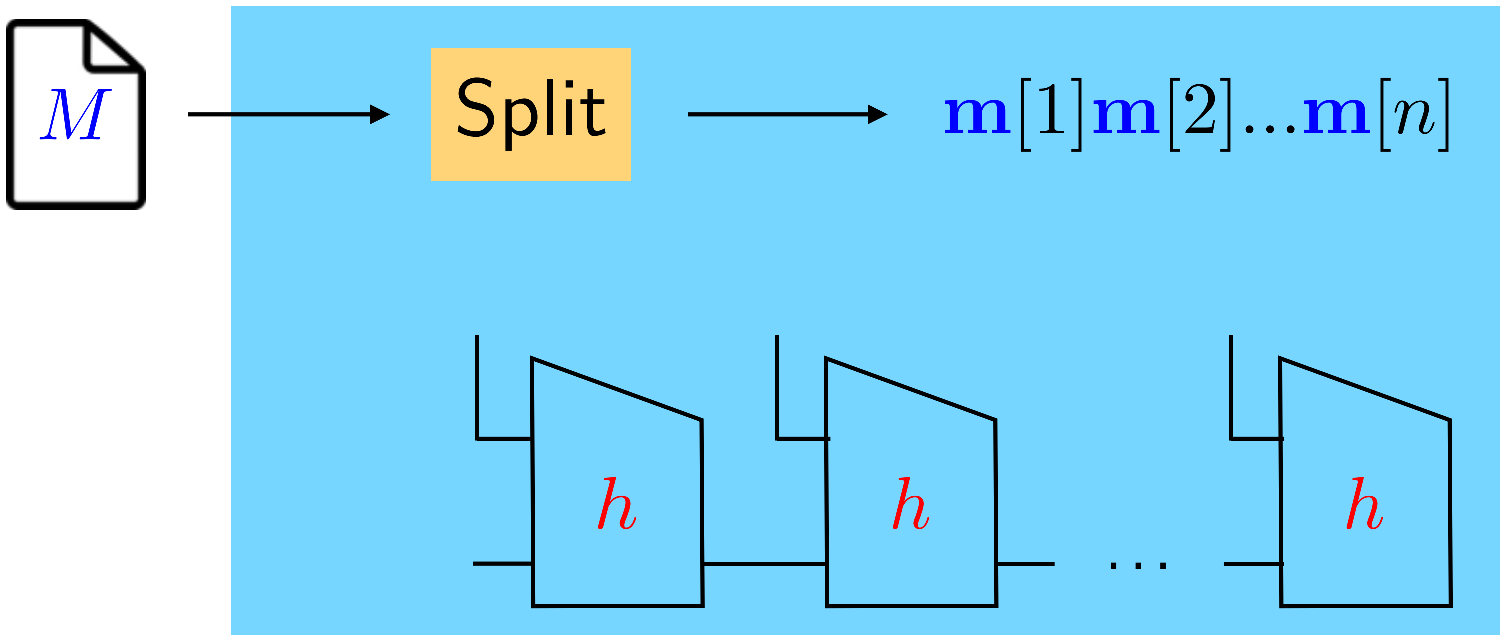

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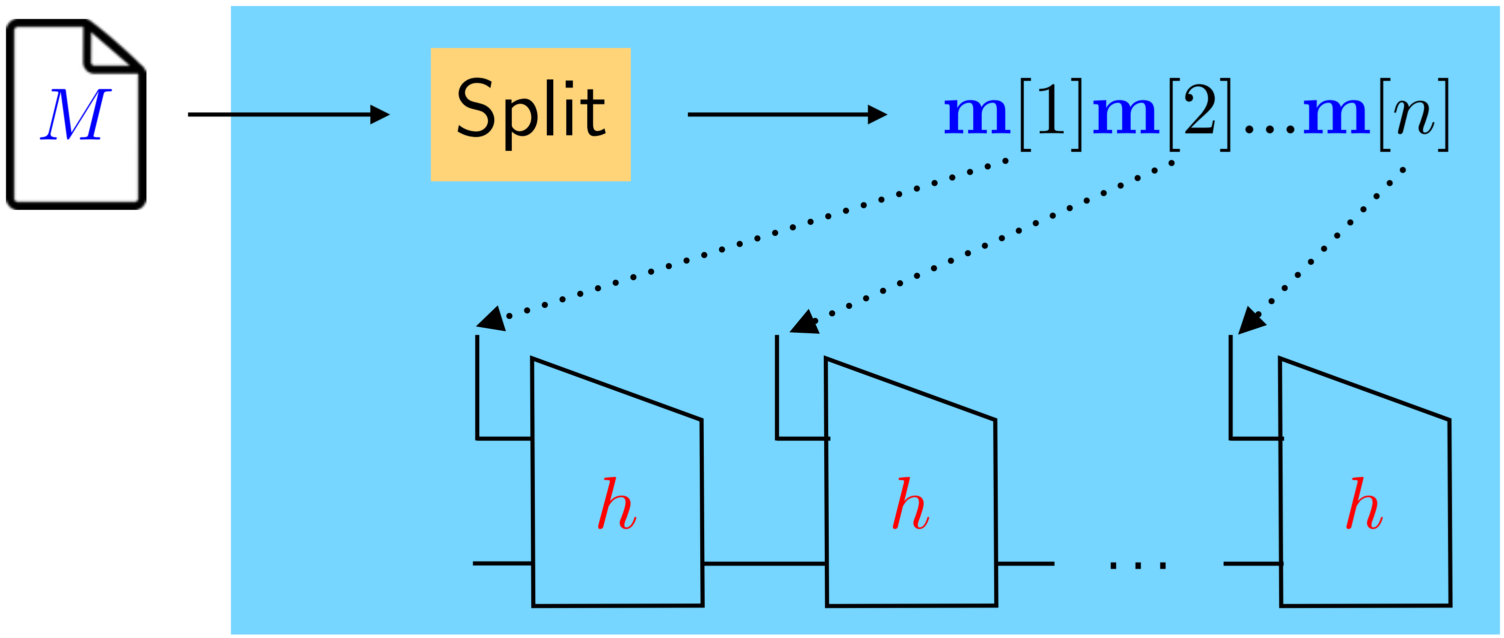

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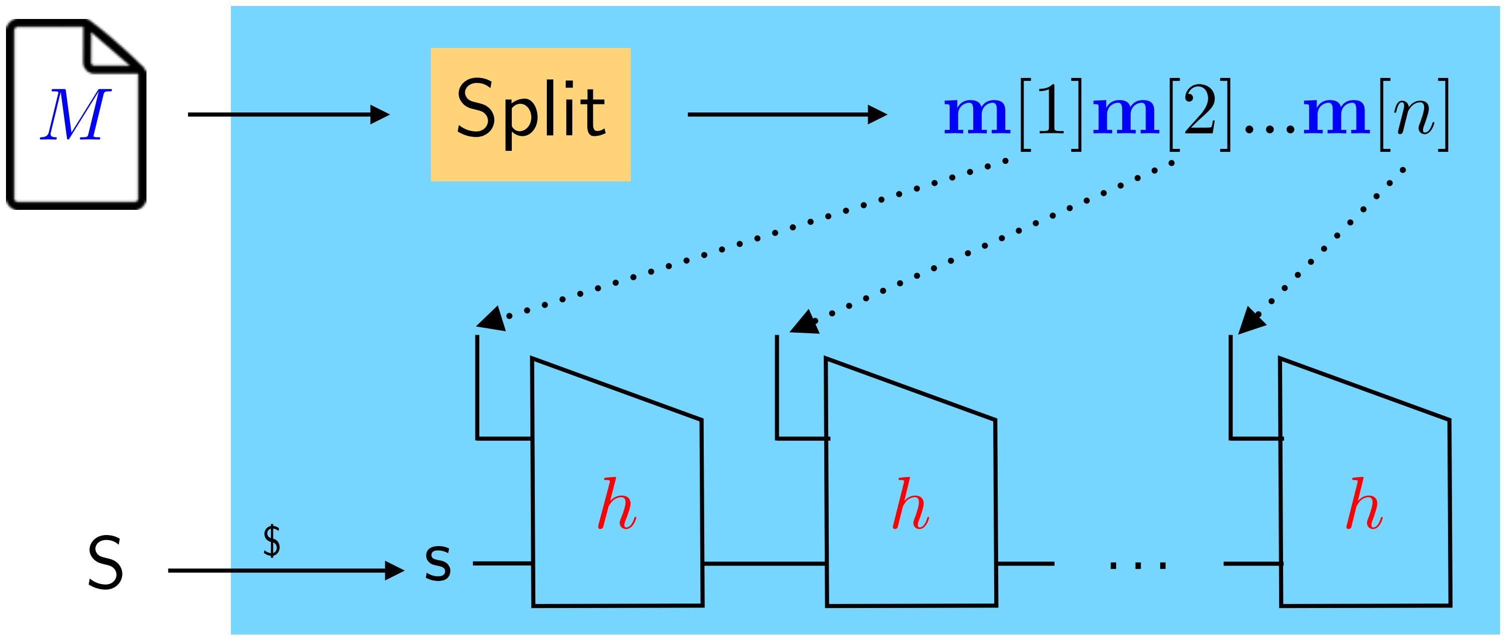

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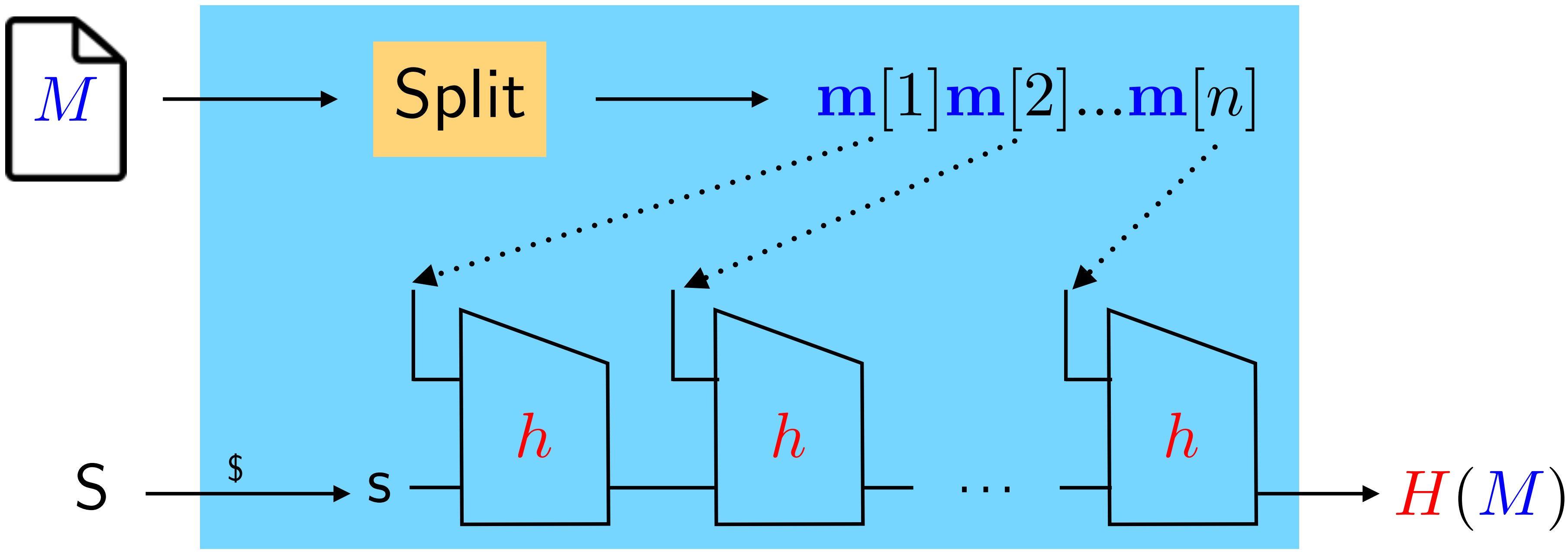
The MD Framework

Splitting function Split : $D \to (\{0,1\}^{h.ml})^*$

Set of starting points $\mathsf{S} \subseteq \{0,1\}^{h.cl}$

$$H = \mathbf{MD}[h, \mathsf{Split}, \mathsf{S}]$$



<table>
<tr><th>H</th><th>h</th><th>Split</th><th>S</th></tr>
<tr><td>MD5</td><td>md5</td><td>M || 1 || 0…0 || ⟨|M|⟩$_{64}$</td><td>{0x67452301 || 0xefcdab89 || 0x98badcfe || 0x10325476}</td></tr>
<tr><td>SHA-1</td><td>sha1</td><td>M || 1 || 0…0 || ⟨|M|⟩$_{64}$</td><td>{0x67452301 || 0xefcdab89 || 0x98badcfe ||0x10325476 || 0xc3d2e1f0}</td></tr>
<tr><td>SHA-256</td><td>sha256</td><td>M || 1 || 0…0 || ⟨|M|⟩$_{64}$</td><td>{0x6a09e667 || 0xbb67ae85 || 0x3c6ef372 || 0xa54ff53a || 0x510e527f || 0x9b05688c || 0x1f83d9ab || 0x5be0cd19}</td></tr>
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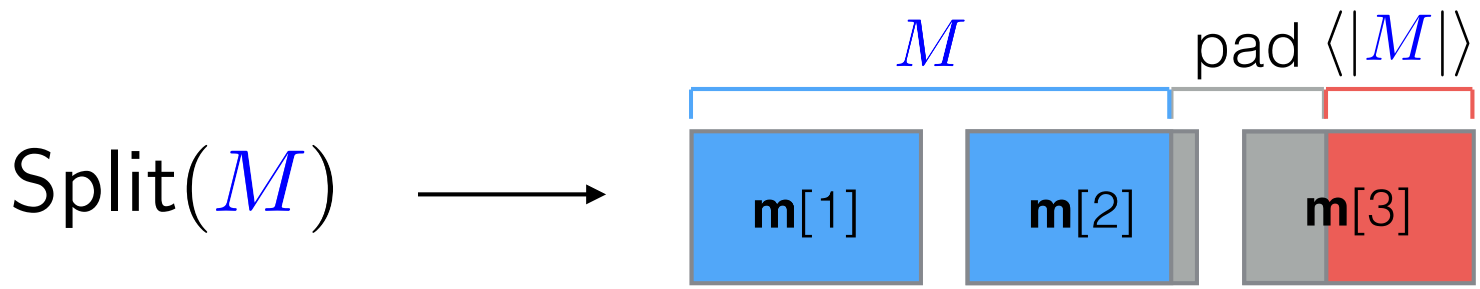
</table>

Possible conditions on Split

Suffix-free

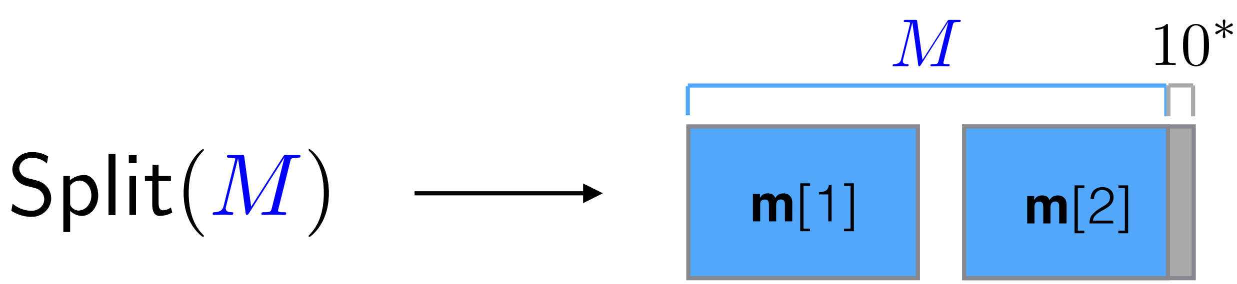
After you apply Split on two distinct messages, neither resulting vector is a suffix of the other.

Typical suffix-free encoding of M (such as in SHA-256):



Injective

After you apply Split on two distinct messages, you get two distinct vectors.



Split(M) is one block shorter, so hashing uses one less call to the compression function. Faster!

	To win, $\mathcal{A}$ must find	such that
CR	$(m_1, c_1) \neq (m_2, c_2)$	$h(m_1, c_1) = h(m_2, c_2)$
CCR	$(m_1, c_1) \neq (m_2, c_2)$ $(m_1', c_1'), (m_2', c_2')$	$h(m_1, c_1) = h(m_2, c_2)$ $c_1 \in \{\mathsf{s}, h(m_1', c_1')\}$ $c_2 \in \{\mathsf{s}, h(m_2', c_2')\}$
Pre	(m, c)	$h(m, c) = \mathsf{s}$

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CCR	$(m_1, c_1) \neq (m_2, c_2)$ $(m'_1, c'_1), (m'_2, c'_2)$	$h(m_1, c_1) = h(m_2, c_2)$ $c_1 \in \{\mathsf{s}, h(m'_1, c'_1)\}$ $c_2 \in \{\mathsf{s}, h(m'_2, c'_2)\}$
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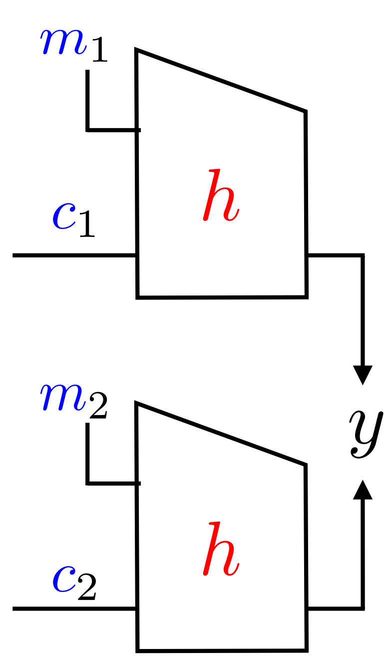

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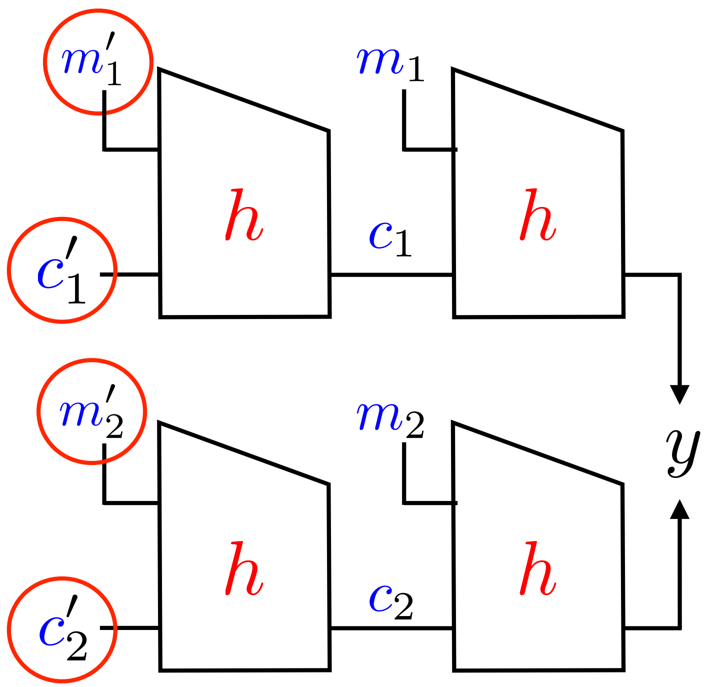

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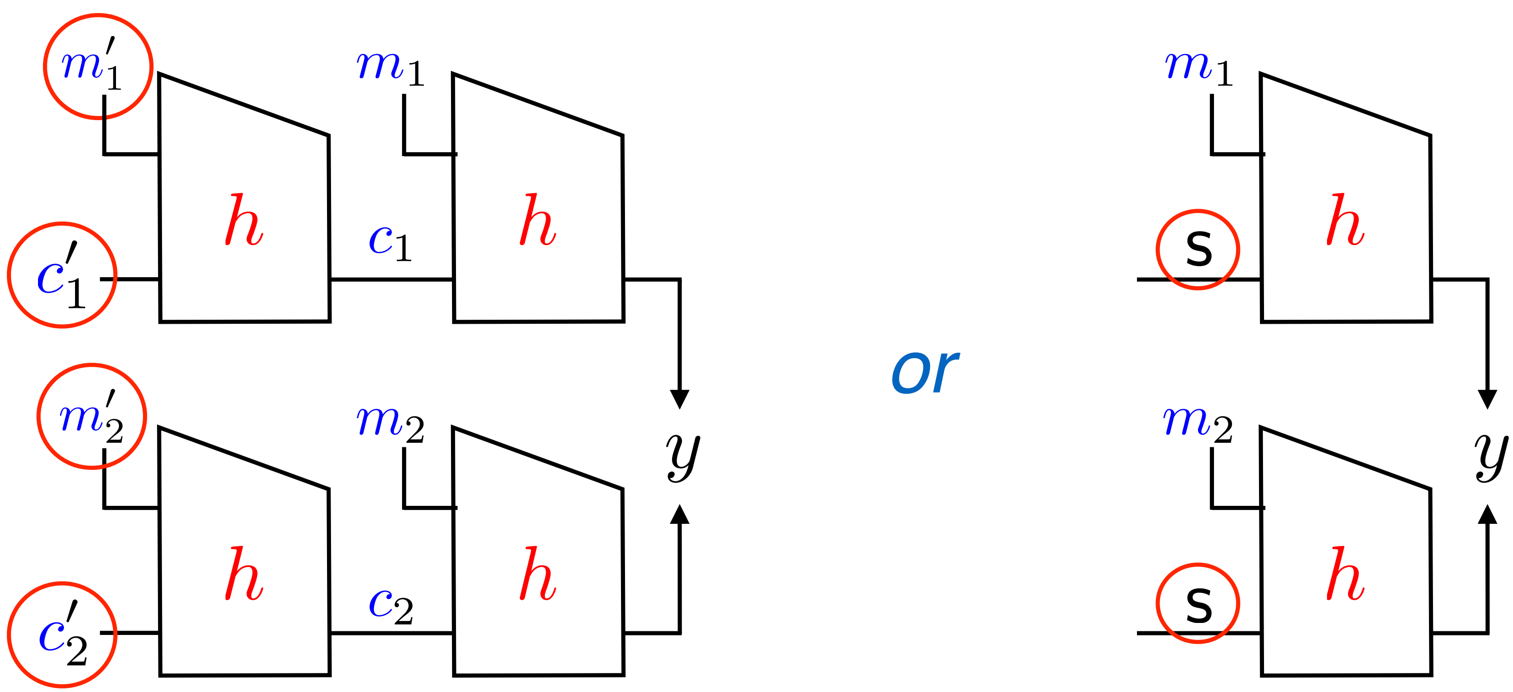

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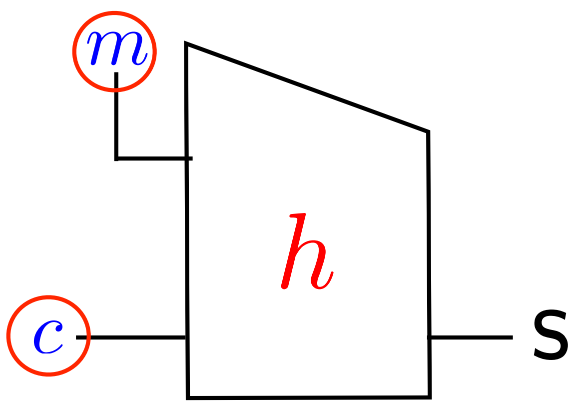

The RS Security Framework

In the previous slide we defined **CR**, **CCR**, and **Pre**.
We give a general definitional framework that yields these and other definitions.

Our definition of security for a compression function h is parameterized by a relation

$$\mathsf{R} : \{0,1\}^* \times \{0,1\}^* \to \{\mathsf{true}, \mathsf{false}\}$$

and a set $\mathsf{S} \subseteq \{0,1\}^*$

Game $\mathrm{G}^{\mathrm{RS}}_{h}(\mathcal{A})$

$\mathrm{s} \leftarrow\!\!\$\; \mathrm{S}\ ;\ out \leftarrow\!\!\$\; \mathcal{A}(\mathrm{s})$

Return $\mathrm{R}(\mathrm{s}, out)$

$\mathsf{R}(\mathsf{s}, out)$

- s: starting value
- out: string that adversary outputs

For $\mathrm{R_{cr}}$ we have $\mathsf{s} = \varepsilon$.

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For R_{cr} we have $\mathsf{s} = \varepsilon$.

R	out	$\mathsf{R}(\mathsf{s}, out)$ **returns true iff**	**Property**
R_{cr}	$((m_1, c_1), (m_2, c_2))$	$h(m_1, c_1) = h(m_2, c_2)$	Collision resistance
R_{ccr}	$((m_1, c_1), (m_2, c_2)),$ $((m_1', c_1'), (m_2', c_2')))$	$\mathsf{R}_{cr}(\varepsilon, ((m_1, c_1), (m_2, c_2)))\wedge$ $(c_1 \in \{\mathsf{s}, h(m_1', c_1')\})\wedge$ $(c_2 \in \{\mathsf{s}, h(m_2', c_2')\})$	Constrained CR
R_{pre}	(m, c)	$h(m, c) = \mathsf{s}$	Pre-image resistance

Results

Typically, $S = \{s\}$ is a singleton set.

	If Split is	and h is	then H = MD[h,Split,S] is	Notes
1	Suffix-free	CR	CR	Known [Me,Da], reproved
2	Suffix-free	CCR	CR	
3	Injective	CCR and Pre	CR	Folklore for CR and Pre [AnSt11]

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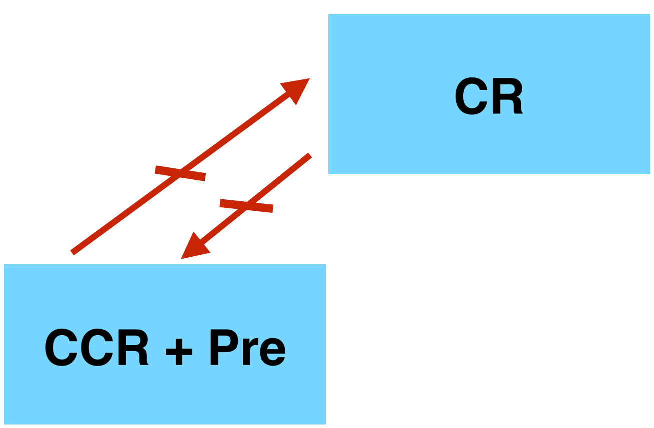

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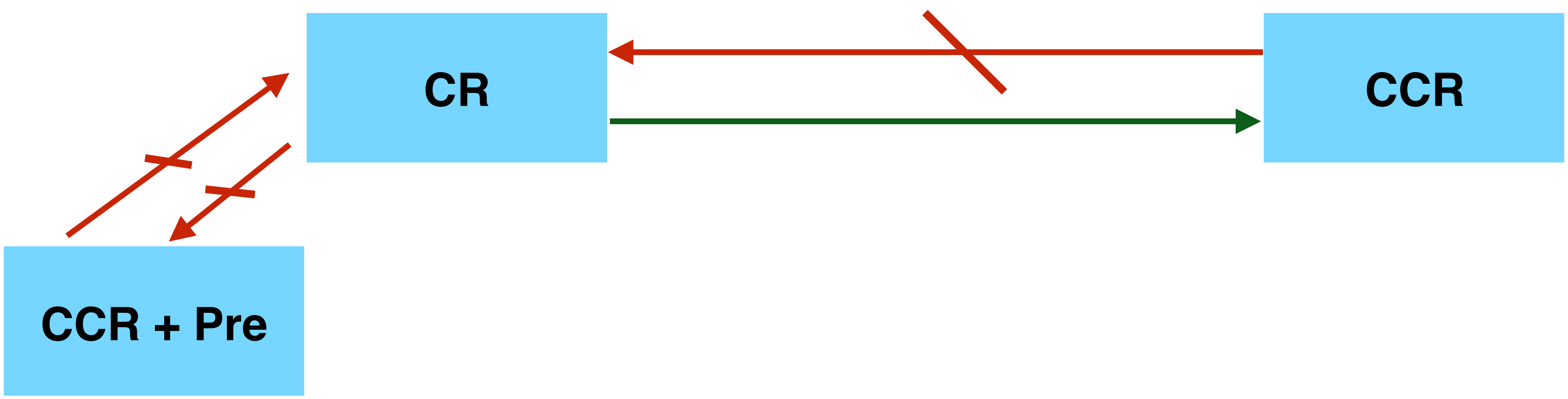

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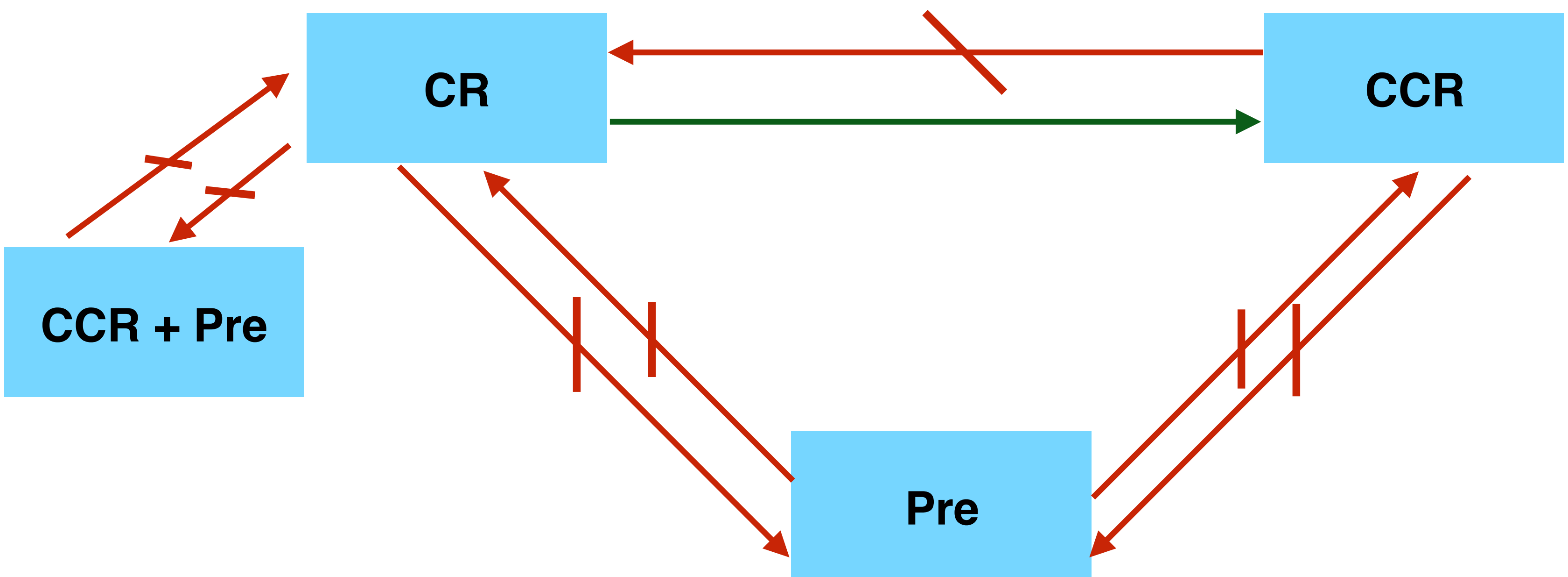

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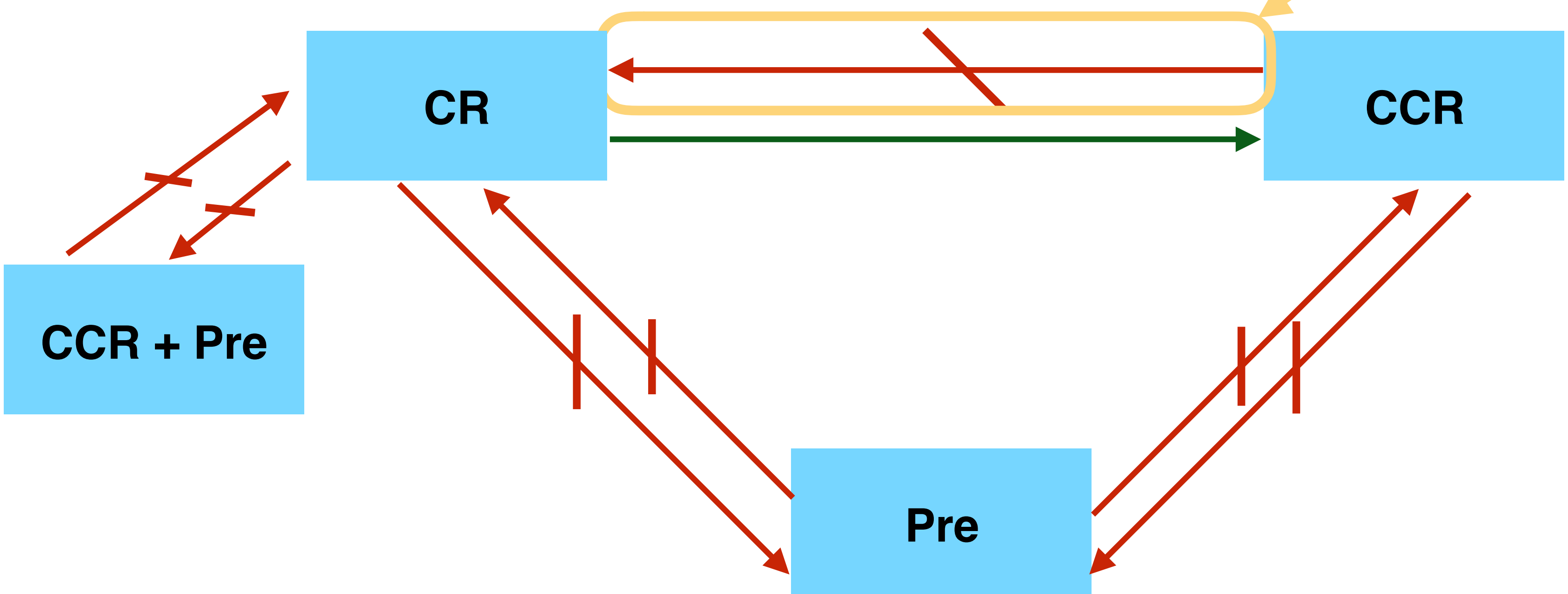
Discussed in the rest of this talk



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$$\mathbf{Adv}_H^{\mathsf{cr}}(\mathcal{A}_H) \leq \mathbf{Adv}_h^{\mathsf{R_{ccr}S}}(\mathcal{A}_h)$$

The time complexity of $\mathcal{A}_h$ is approximately that of $\mathcal{A}_H$ plus the time to compute H. The memory complexity of $\mathcal{A}_h$ is the maximum of the memory complexity of $\mathcal{A}_H$ and term linear in the length of the output of $\mathcal{A}_H$.

```
adversary A_h(s)
(M_1, M_2) ← A_H(s, ε)
m_1 ← Split(M_1) ; m_2 ← Split(M_2) ; n_1 ← |m_1| ; n_2 ← |m_2|
c_1[1] ← s ; c_2[1] ← s
For i = 1, ..., n_1 do c_1[i + 1] ← h(m_1[i], c_1[i])
For i = 1, ..., n_2 do c_2[i + 1] ← h(m_2[i], c_2[i])
b ← argmin_d(n_d)
For i = 0, ..., n_b − 2 do
    (m_1, c_1) ← (m_1[n_1 − i], c_1[n_1 − i])
    (m_2, c_2) ← (m_2[n_2 − i], c_2[n_2 − i])
    a_1 ← (m_1[n_1 − i − 1], c_1[n_1 − i − 1])
    a_2 ← (m_2[n_2 − i − 1], c_2[n_2 − i − 1])
    If (m_1, c_1) ≠ (m_2, c_2) then
        Return ((m_1, c_1), (m_2, c_2), a_1, a_2)
If n_1 = n_2 then
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    a_1 ← 1; a_2 ← 2
    Return ((m_1, c_1), (m_2, c_2), a_1, a_2)
(m_1, c_1) ← (m_1[n_1 − n_b + 1], c_1[n_1 − n_b + 1])
(m_2, c_2) ← (m_2[n_2 − n_b + 1], c_2[n_2 − n_b + 1])
a_{3−b} ← (m_{3−b}[n_{3−b} − n_b], c_{3−b}[n_{3−b} − n_b])
a_b ← a_{3−b}
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Proof uses the back-tracking paradigm of [Me,Da] but constructs a CCR-violating adversary rather than a CR-violating one.

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adversary $\mathcal{A}_h(\mathrm{s})$

$(M_1, M_2) \leftarrow \mathcal{A}_H(\mathrm{s}, \varepsilon)$

$\mathbf{m}_1 \leftarrow \mathrm{Split}(M_1)$; $\mathbf{m}_2 \leftarrow \mathrm{Split}(M_2)$; $n_1 \leftarrow |\mathbf{m}_1|$; $n_2 \leftarrow |\mathbf{m}_2|$

$\mathbf{c}_1[1] \leftarrow \mathrm{s}$; $\mathbf{c}_2[1] \leftarrow \mathrm{s}$

For $i = 1, \ldots, n_1$ do $\mathbf{c}_1[i+1] \leftarrow h(\mathbf{m}_1[i], \mathbf{c}_1[i])$

For $i = 1, \ldots, n_2$ do $\mathbf{c}_2[i+1] \leftarrow h(\mathbf{m}_2[i], \mathbf{c}_2[i])$

$b \leftarrow \mathrm{argmin}_d(n_d)$

For $i = 0, \ldots, n_b - 2$ do

    $(m_1, c_1) \leftarrow (\mathbf{m}_1[n_1 - i], \mathbf{c}_1[n_1 - i])$

    $(m_2, c_2) \leftarrow (\mathbf{m}_2[n_2 - i], \mathbf{c}_2[n_2 - i])$

    $a_1 \leftarrow (\mathbf{m}_1[n_1 - i - 1], \mathbf{c}_1[n_1 - i - 1])$

    $a_2 \leftarrow (\mathbf{m}_2[n_2 - i - 1], \mathbf{c}_2[n_2 - i - 1])$

    If $(m_1, c_1) \neq (m_2, c_2)$ then

        Return $((m_1, c_1), (m_2, c_2), a_1, a_2)$

If $n_1 = n_2$ then

    $(m_1, c_1) \leftarrow (\mathbf{m}_1[1], \mathbf{c}_1[1])$; $(m_2, c_2) \leftarrow (\mathbf{m}_2[1], \mathbf{c}_2[1])$

    $a_1 \leftarrow 1$; $a_2 \leftarrow 2$

    Return $((m_1, c_1), (m_2, c_2), a_1, a_2)$

$(m_1, c_1) \leftarrow (\mathbf{m}_1[n_1 - n_b + 1], \mathbf{c}_1[n_1 - n_b + 1])$

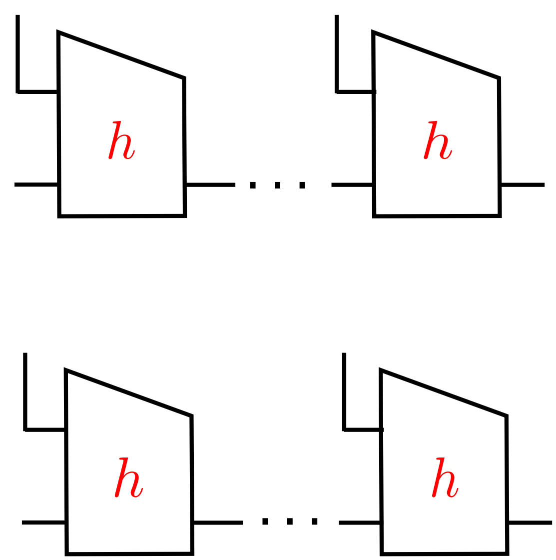
$(m_2, c_2) \leftarrow (\mathbf{m}_2[n_2 - n_b + 1], \mathbf{c}_2[n_2 - n_b + 1])$

$a_{3-b} \leftarrow (\mathbf{m}_{3-b}[n_{3-b} - n_b], \mathbf{c}_{3-b}[n_{3-b} - n_b])$

$a_b \leftarrow a_{3-b}$

Return $((m_1, c_1), (m_2, c_2), a_1, a_2)$

Proof uses the back-tracking paradigm of [Me,Da] but constructs a CCR-violating adversary rather than a CR-violating one.



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$\mathbf{c}_1[1] \leftarrow s$; $\mathbf{c}_2[1] \leftarrow s$
For $i = 1, \ldots, n_1$ do $\mathbf{c}_1[i+1] \leftarrow h(\mathbf{m}_1[i], \mathbf{c}_1[i])$
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$(m_2, c_2) \leftarrow (\mathbf{m}_2[n_2 - n_b + 1], \mathbf{c}_2[n_2 - n_b + 1])$
$a_{3-b} \leftarrow (\mathbf{m}_{3-b}[n_{3-b} - n_b], \mathbf{c}_{3-b}[n_{3-b} - n_b])$
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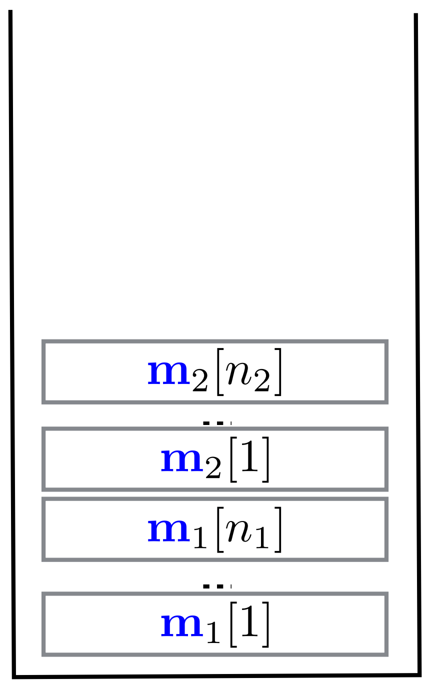

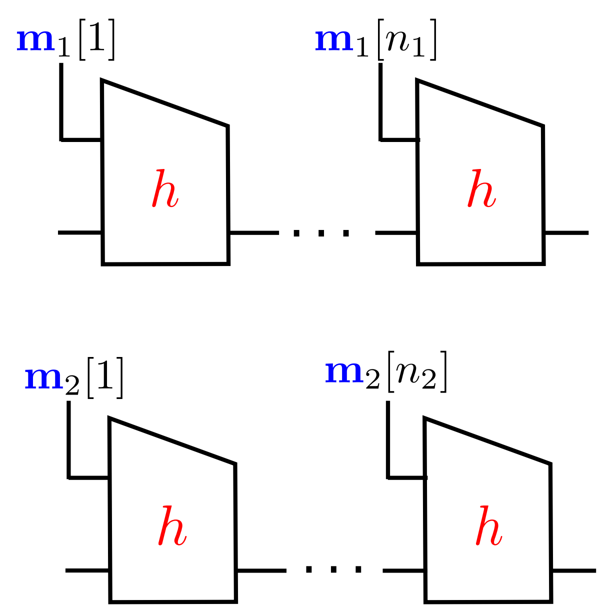

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$(m_1, c_1) \leftarrow (\mathbf{m}_1[n_1 - n_b + 1], \mathbf{c}_1[n_1 - n_b + 1])$
$(m_2, c_2) \leftarrow (\mathbf{m}_2[n_2 - n_b + 1], \mathbf{c}_2[n_2 - n_b + 1])$
$a_{3-b} \leftarrow (\mathbf{m}_{3-b}[n_{3-b} - n_b], \mathbf{c}_{3-b}[n_{3-b} - n_b])$
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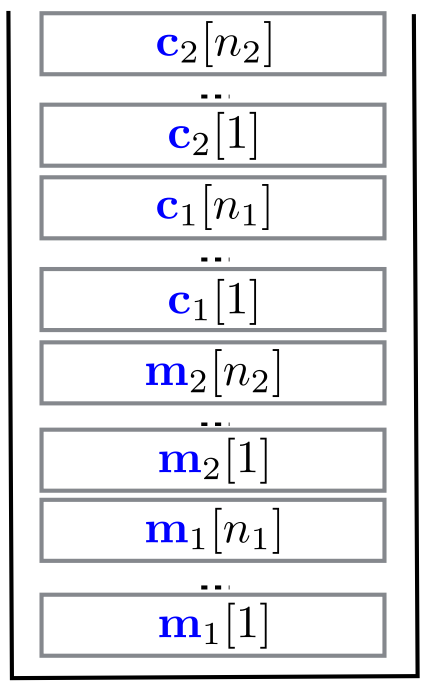

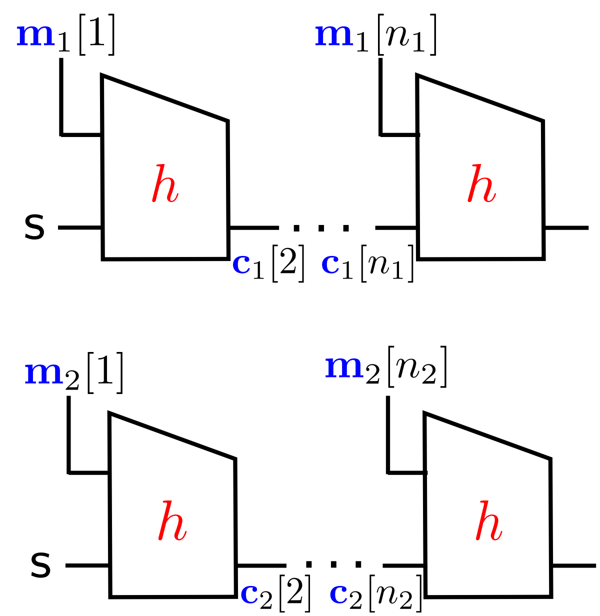

Theorem

Let Split be a suffix-free splitting function. Given an adversary $\mathcal{A}_H$, we define $\mathcal{A}_h$ such that

$$\mathbf{Adv}^{\mathsf{cr}}_{H}(\mathcal{A}_H) \leq \mathbf{Adv}^{\mathsf{R_{ccr}S}}_{h}(\mathcal{A}_h)$$

The time complexity of $\mathcal{A}_h$ is approximately that of $\mathcal{A}_H$ plus the time to compute H. The memory complexity of $\mathcal{A}_h$ is the maximum of the memory complexity of $\mathcal{A}_H$ and term linear in the length of the output of $\mathcal{A}_H$.

adversary $\mathcal{A}_h(\mathrm{s})$

$(M_1, M_2) \leftarrow \mathcal{A}_H(\mathrm{s}, \varepsilon)$

$\mathbf{m}_1 \leftarrow \mathrm{Split}(M_1)$; $\mathbf{m}_2 \leftarrow \mathrm{Split}(M_2)$; $n_1 \leftarrow |\mathbf{m}_1|$; $n_2 \leftarrow |\mathbf{m}_2|$

$\mathbf{c}_1[1] \leftarrow \mathrm{s}$; $\mathbf{c}_2[1] \leftarrow \mathrm{s}$

For $i = 1, \ldots, n_1$ do $\mathbf{c}_1[i+1] \leftarrow h(\mathbf{m}_1[i], \mathbf{c}_1[i])$

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  If $(m_1, c_1) \neq (m_2, c_2)$ then

    Return $((m_1, c_1), (m_2, c_2), a_1, a_2)$

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  $a_1 \leftarrow 1$; $a_2 \leftarrow 2$

  Return $((m_1, c_1), (m_2, c_2), a_1, a_2)$

$(m_1, c_1) \leftarrow (\mathbf{m}_1[n_1 - n_b + 1], \mathbf{c}_1[n_1 - n_b + 1])$

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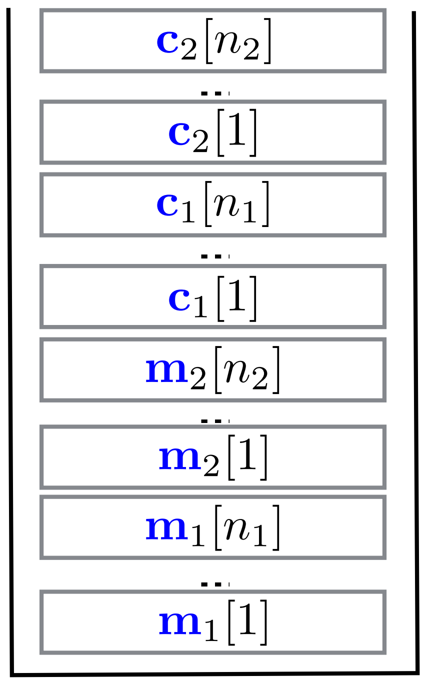

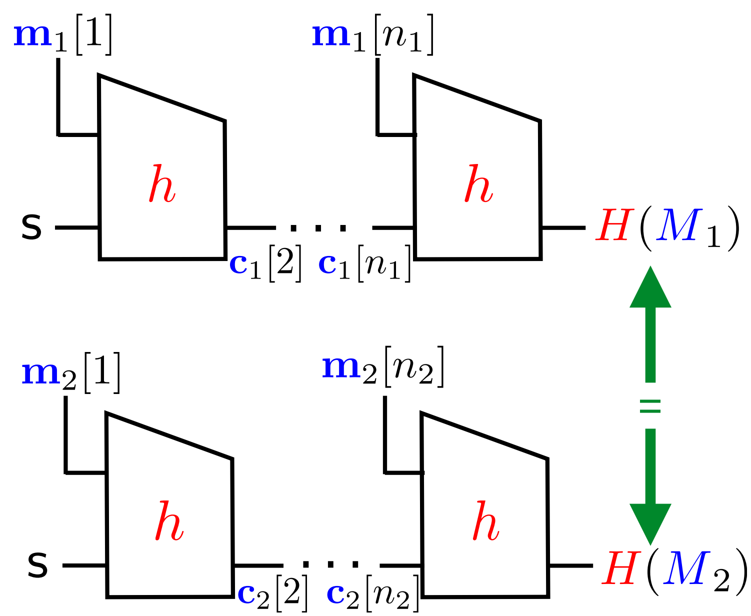

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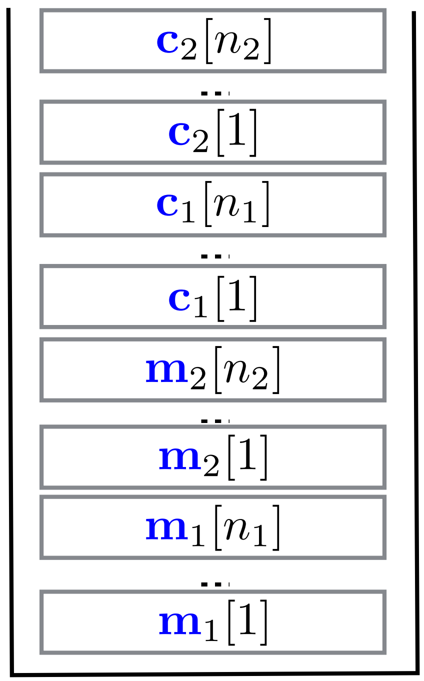

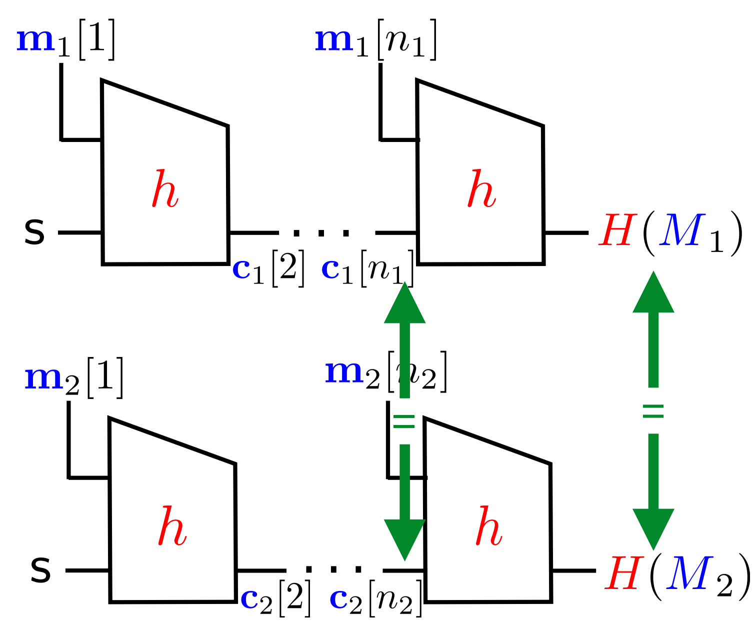

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  $a_1 \leftarrow (\mathbf{m}_1[n_1 - i - 1], \mathbf{c}_1[n_1 - i - 1])$

  $a_2 \leftarrow (\mathbf{m}_2[n_2 - i - 1], \mathbf{c}_2[n_2 - i - 1])$

  If $(m_1, c_1) \neq (m_2, c_2)$ then

    Return $((m_1, c_1), (m_2, c_2), a_1, a_2)$

If $n_1 = n_2$ then

  $(m_1, c_1) \leftarrow (\mathbf{m}_1[1], \mathbf{c}_1[1])$; $(m_2, c_2) \leftarrow (\mathbf{m}_2[1], \mathbf{c}_2[1])$

  $a_1 \leftarrow 1; a_2 \leftarrow 2$

  Return $((m_1, c_1), (m_2, c_2), a_1, a_2)$

$(m_1, c_1) \leftarrow (\mathbf{m}_1[n_1 - n_b + 1], \mathbf{c}_1[n_1 - n_b + 1])$

$(m_2, c_2) \leftarrow (\mathbf{m}_2[n_2 - n_b + 1], \mathbf{c}_2[n_2 - n_b + 1])$

$a_{3-b} \leftarrow (\mathbf{m}_{3-b}[n_{3-b} - n_b], \mathbf{c}_{3-b}[n_{3-b} - n_b])$

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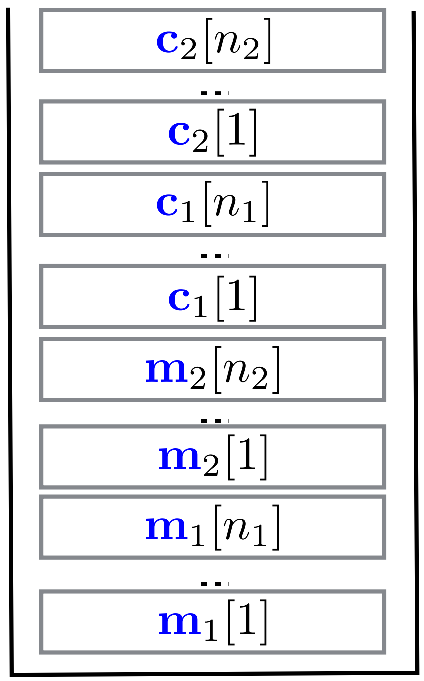

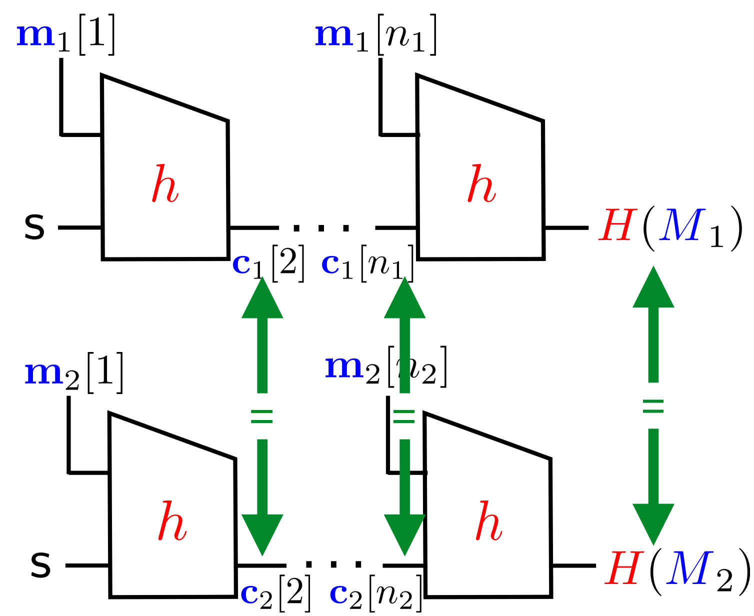

Closer look at memory complexity

Theorem Same as above, except:
The memory complexity of $\mathcal{A}_h$ is the maximum of the memory complexity of $\mathcal{A}_H$ and a **small constant**.

```
adversary A_h(s)
(M_1, M_2) ← A_H(s, ε)
m_1 ← Split(M_1) ; m_2 ← Split(M_2) ; n_1 ← |m_1| ; n_2 ← |m_2|
c_1[1] ← s ; c_2[1] ← s; n ← min(n_1, n_2)
If (n_1 > n_2) then
   For i = 1, ..., n_1 − n_2 do c_1[i + 1] ← h(m_1[i], c_1[i])
If (n_2 > n_1) then
   For i = 1, ..., n_2 − n_1 do c_2[i + 1] ← h(m_2[i], c_2[i])
For i = 1, ..., n do
   m_1 ← m_1[n_1 − n + i]; c_1 ← c_1[n_1 − n + i]
   m_2 ← m_2[n_2 − n + i]; c_2 ← c_2[n_2 − n + i]
   c'_1 ← h(m_1, c_1)
   c'_2 ← h(m_2, c_2)
   If (c'_1 = c'_2) and (m_1, c_1) ≠ (m_2, c_2) then
      a_1 ← (m_1[n_1 − n + i − 1], c_1[n_1 − n + i − 1])
      a_2 ← (m_2[n_2 − n + i − 1], c_2[n_2 − n + i − 1])
      Return ((m_1, c_1), (m_2, c_2), a_1, a_2)
   c_1[n_1 − n + i + 1] ← c'_1
   c_2[n_2 − n + i + 1] ← c'_2
Return ⊥
```

ACFK17: “memory tightness is important”

Natural reduction was *not* memory tight.

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adversary $\mathcal{A}_h(s)$

$(M_1, M_2) \leftarrow \mathcal{A}_H(s, \epsilon)$

$\mathbf{m}_1 \leftarrow \mathsf{Split}(M_1)$; $\mathbf{m}_2 \leftarrow \mathsf{Split}(M_2)$; $n_1 \leftarrow |\mathbf{m}_1|$; $n_2 \leftarrow |\mathbf{m}_2|$

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  $c_1' \leftarrow h(m_1, c_1)$

  $c_2' \leftarrow h(m_2, c_2)$

  If $(c_1' = c_2')$ and $(m_1, c_1) \neq (m_2, c_2)$ then

    $a_1 \leftarrow (\mathbf{m}_1[n_1 - n + i - 1], \mathbf{c}_1[n_1 - n + i - 1])$

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    Return $((m_1, c_1), (m_2, c_2), a_1, a_2)$

  $\mathbf{c}_1[n_1 - n + i + 1] \leftarrow c_1'$

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Return $\perp$

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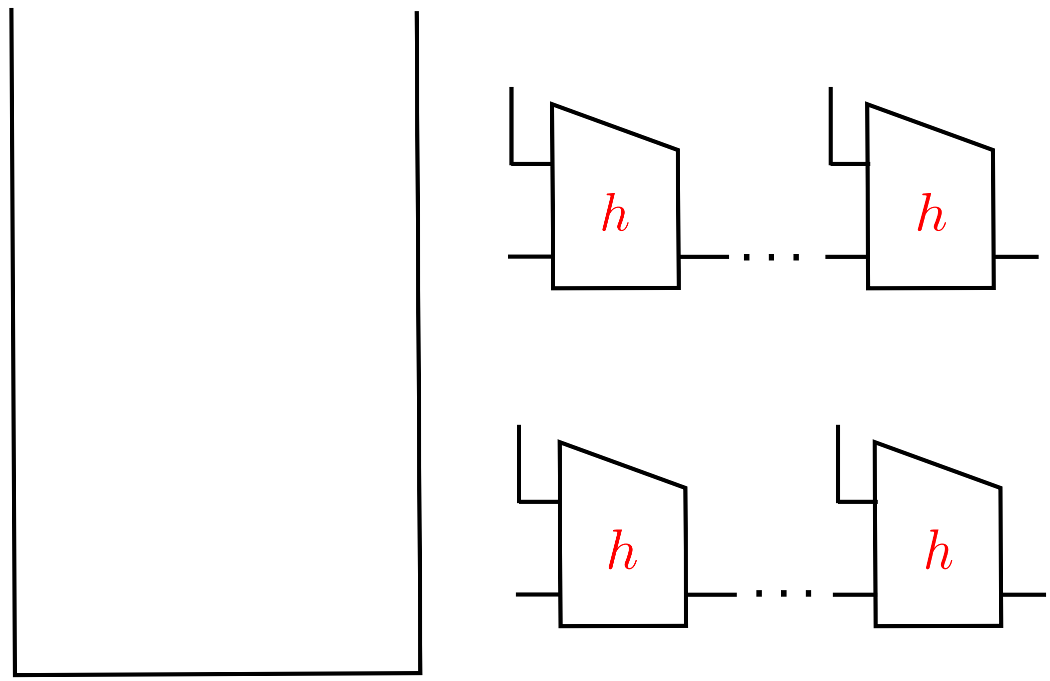

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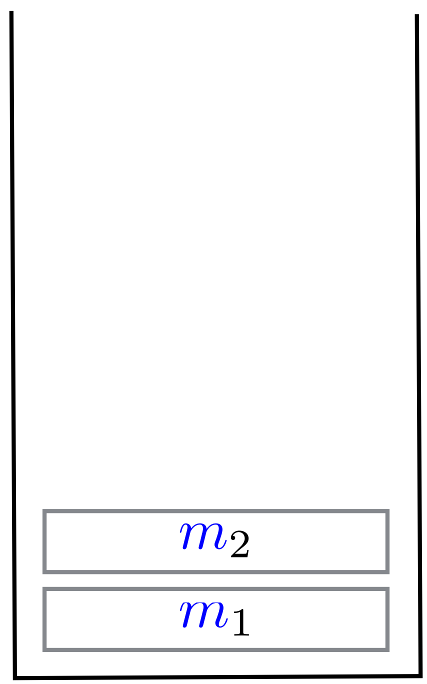

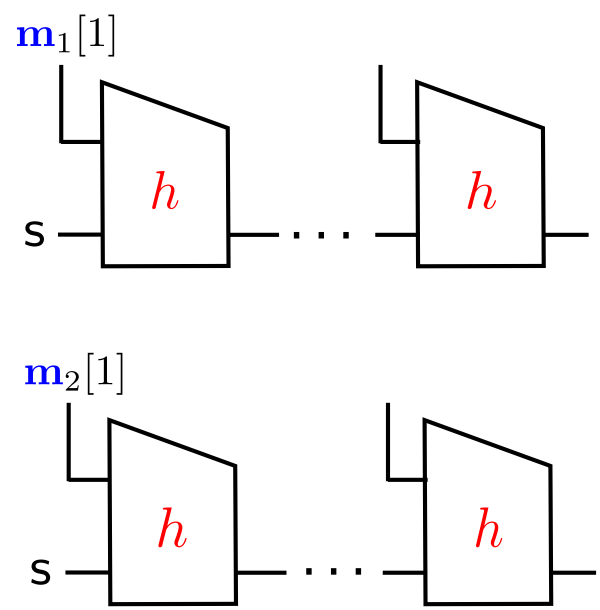

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    Return $((m_1, c_1), (m_2, c_2), a_1, a_2)$

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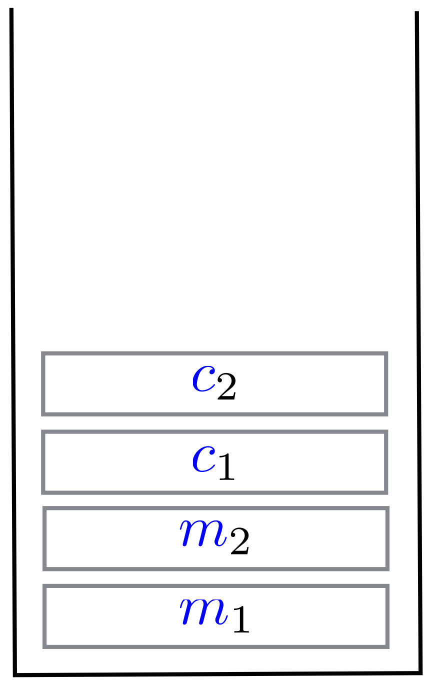

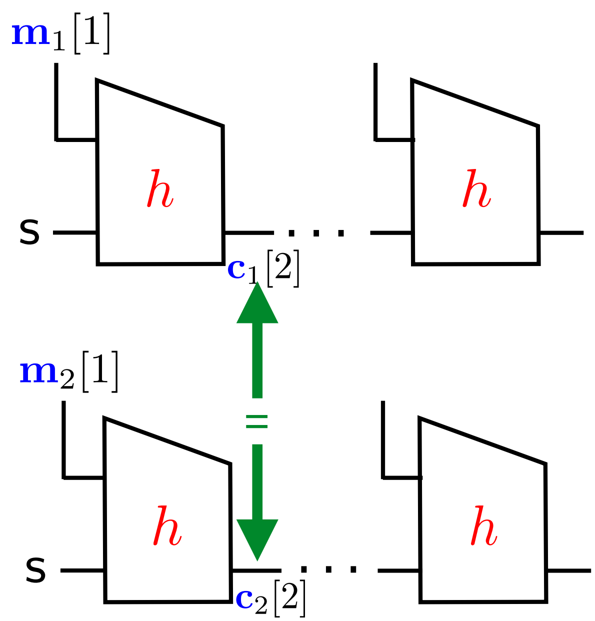

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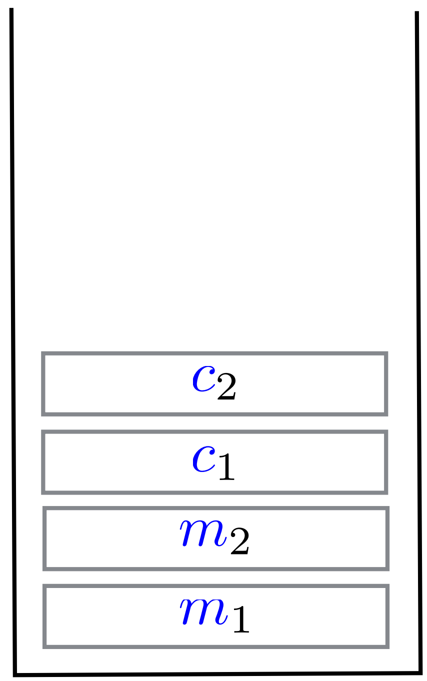

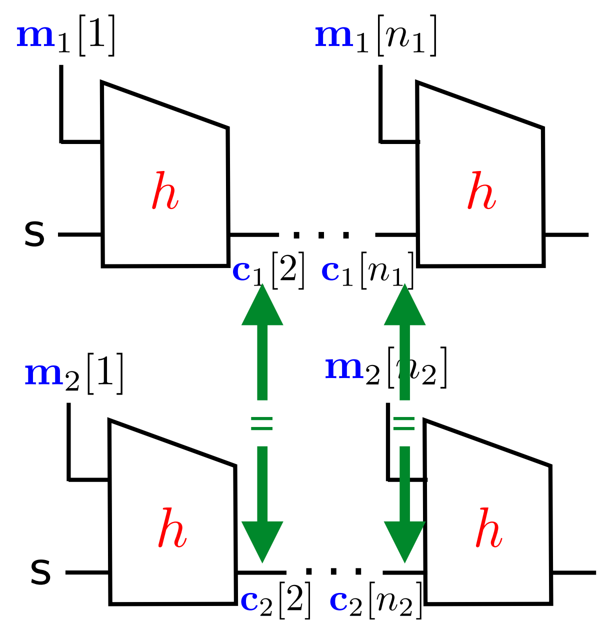

Closer look at memory complexity

Theorem Same as above, except:
The memory complexity of $\mathcal{A}_h$ is the maximum of the memory complexity of $\mathcal{A}_H$ and a **small constant**.

adversary $\mathcal{A}_h(s)$

$(M_1, M_2) \leftarrow \mathcal{A}_H(s, \epsilon)$
$\mathbf{m}_1 \leftarrow \mathsf{Split}(M_1)$; $\mathbf{m}_2 \leftarrow \mathsf{Split}(M_2)$; $n_1 \leftarrow |\mathbf{m}_1|$; $n_2 \leftarrow |\mathbf{m}_2|$
$\mathbf{c}_1[1] \leftarrow s$; $\mathbf{c}_2[1] \leftarrow s$; $n \leftarrow \min(n_1, n_2)$
If $(n_1 > n_2)$ then
  For $i = 1, \ldots, n_1 - n_2$ do $\mathbf{c}_1[i+1] \leftarrow h(\mathbf{m}_1[i], \mathbf{c}_1[i])$
If $(n_2 > n_1)$ then
  For $i = 1, \ldots, n_2 - n_1$ do $\mathbf{c}_2[i+1] \leftarrow h(\mathbf{m}_2[i], \mathbf{c}_2[i])$
For $i = 1, \ldots, n$ do
  $m_1 \leftarrow \mathbf{m}_1[n_1 - n + i]$; $c_1 \leftarrow \mathbf{c}_1[n_1 - n + i]$
  $m_2 \leftarrow \mathbf{m}_2[n_2 - n + i]$; $c_2 \leftarrow \mathbf{c}_2[n_2 - n + i]$
  $c_1' \leftarrow h(m_1, c_1)$
  $c_2' \leftarrow h(m_2, c_2)$
  If $(c_1' = c_2')$ and $(m_1, c_1) \neq (m_2, c_2)$ then
    $a_1 \leftarrow (\mathbf{m}_1[n_1 - n + i - 1], \mathbf{c}_1[n_1 - n + i - 1])$
    $a_2 \leftarrow (\mathbf{m}_2[n_2 - n + i - 1], \mathbf{c}_2[n_2 - n + i - 1])$
    Return $((m_1, c_1), (m_2, c_2), a_1, a_2)$
  $\mathbf{c}_1[n_1 - n + i + 1] \leftarrow c_1'$
  $\mathbf{c}_2[n_2 - n + i + 1] \leftarrow c_2'$
Return $\perp$

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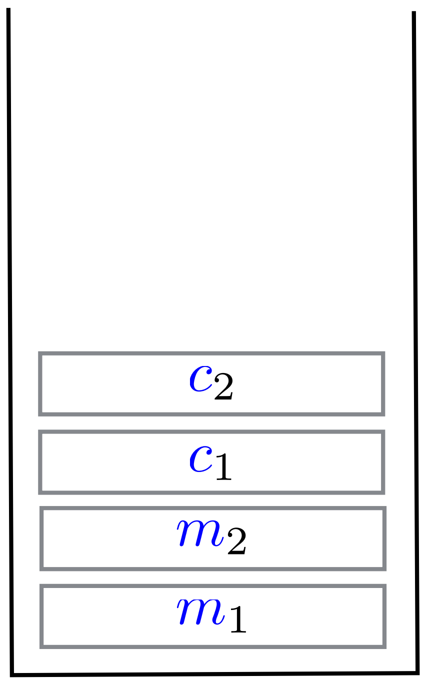

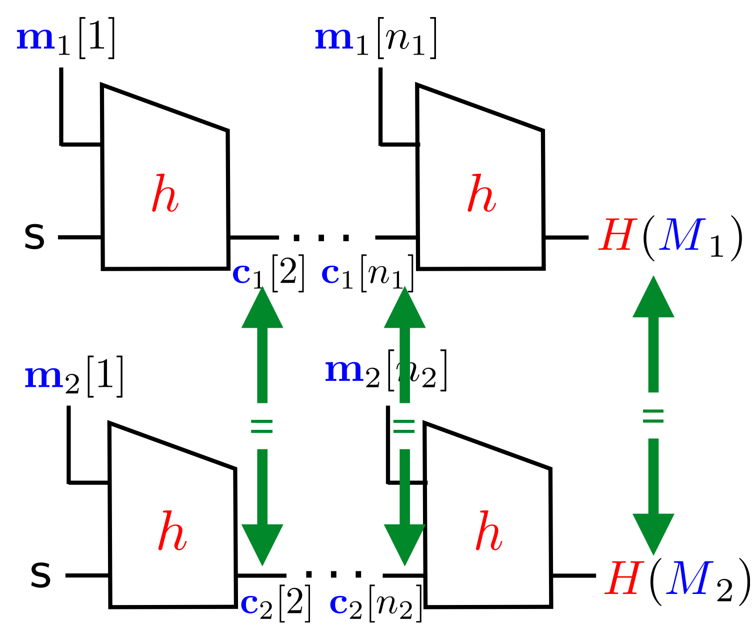

CCR is strictly weaker than CR

We show this by defining a CCR but not CR secure compression function:

$$h : \{0,1\}^{h.ml} \times \{0,1\}^{h.cl} \to \{0,1\}^{h.cl}$$

Assumptions

1. Split is suffix-free
2. h has access to a CR function $h' : \{0,1\}^{h.ml} \times \{0,1\}^{h.cl} \to \{0,1\}^{h.cl-1}$
3. $\mathsf{S} = \{0,1\}^{h.cl} \setminus \{1\|0^{h.cl-1}, 1^2\|0^{h.cl-2}\}$

Claims

1. h is CCR
2. h is **not** CR
3. $H = \mathrm{MD}[h, \mathsf{Split}, \mathsf{S}]$ **is** CR

$h(m, c)$

If $(m, c) \in \{(0^{h.ml}, 1 \| 0^{h.cl-1}), (1^{h.ml}, 1^2 \| 0^{h.cl-2})\}$
    Return $1^{h.cl}$
Return $0 \| h'(m, c)$

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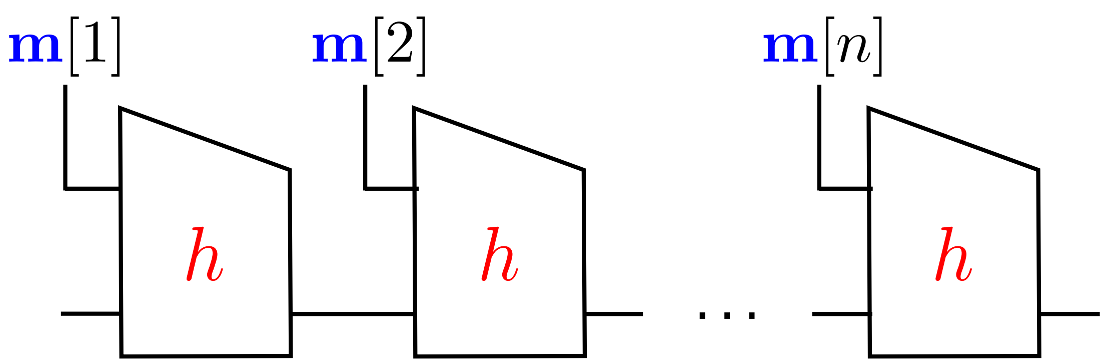

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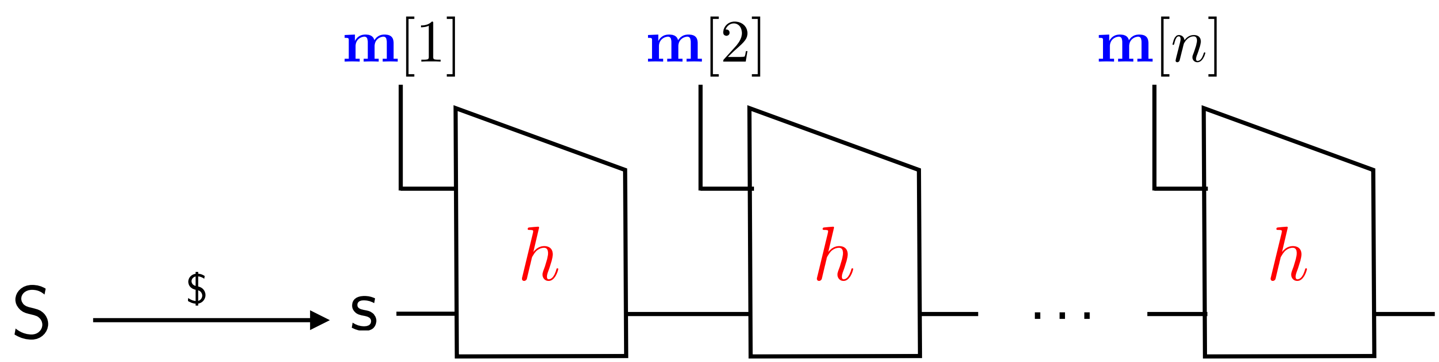

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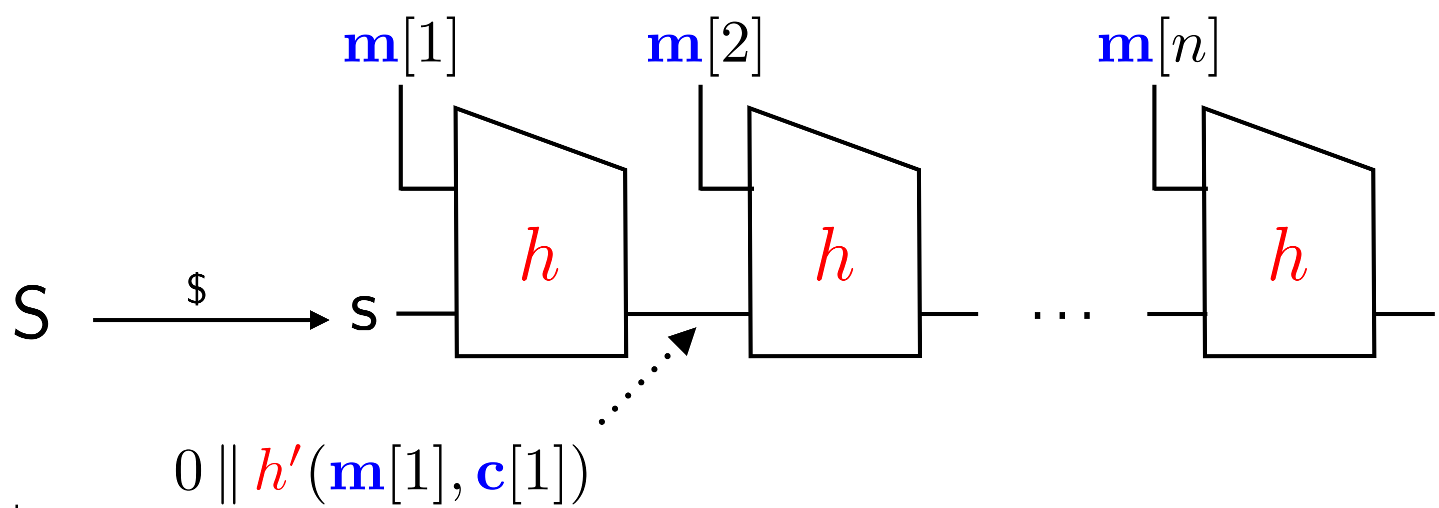

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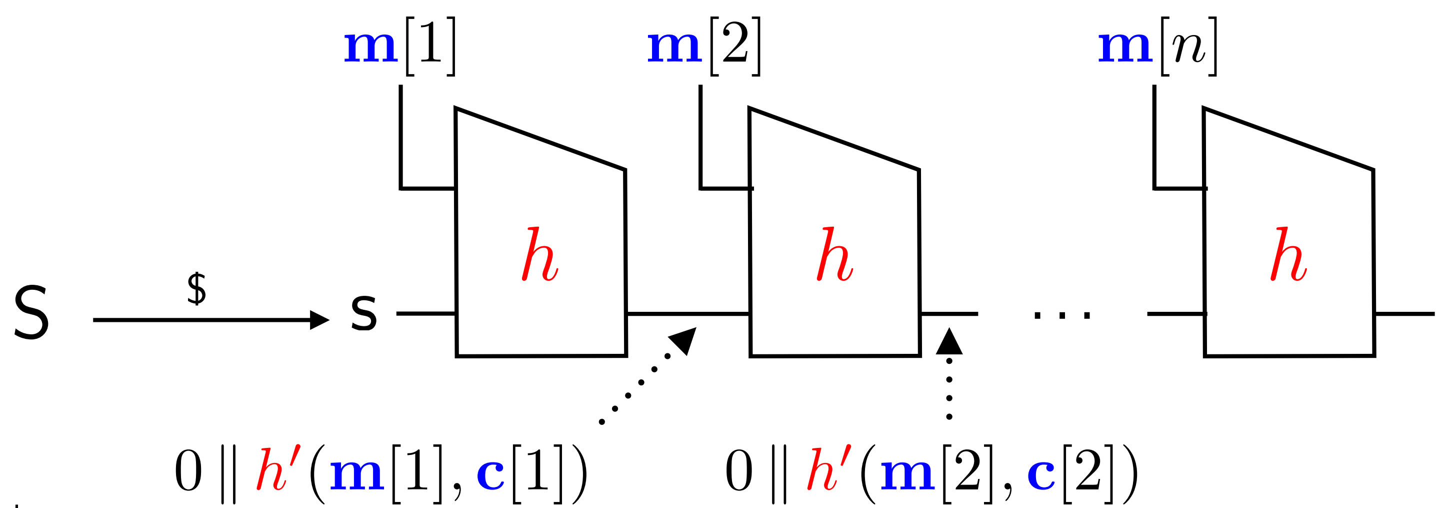

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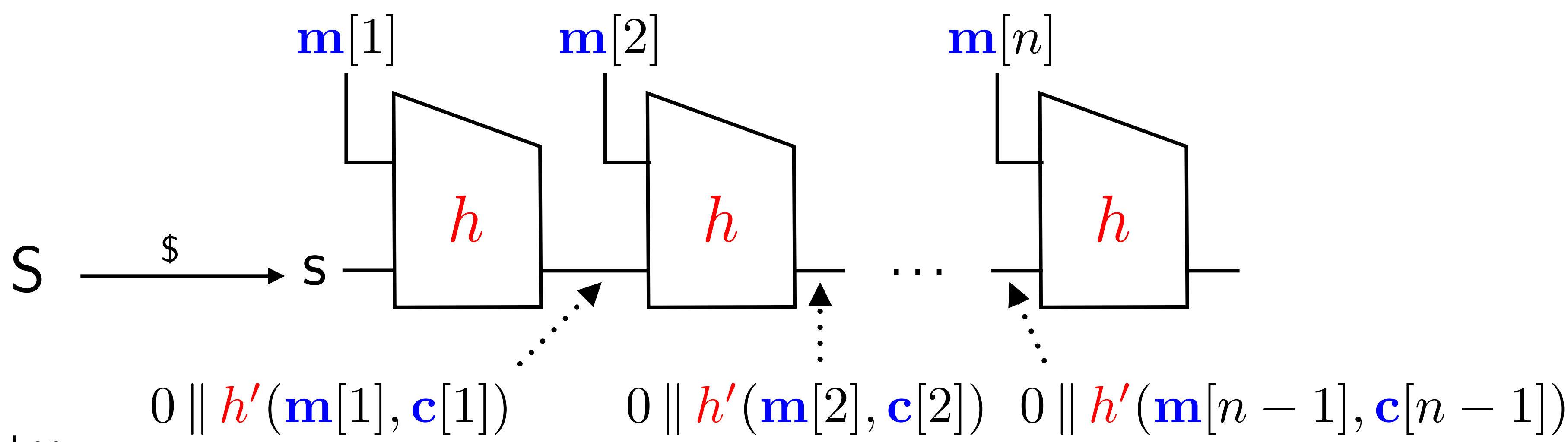

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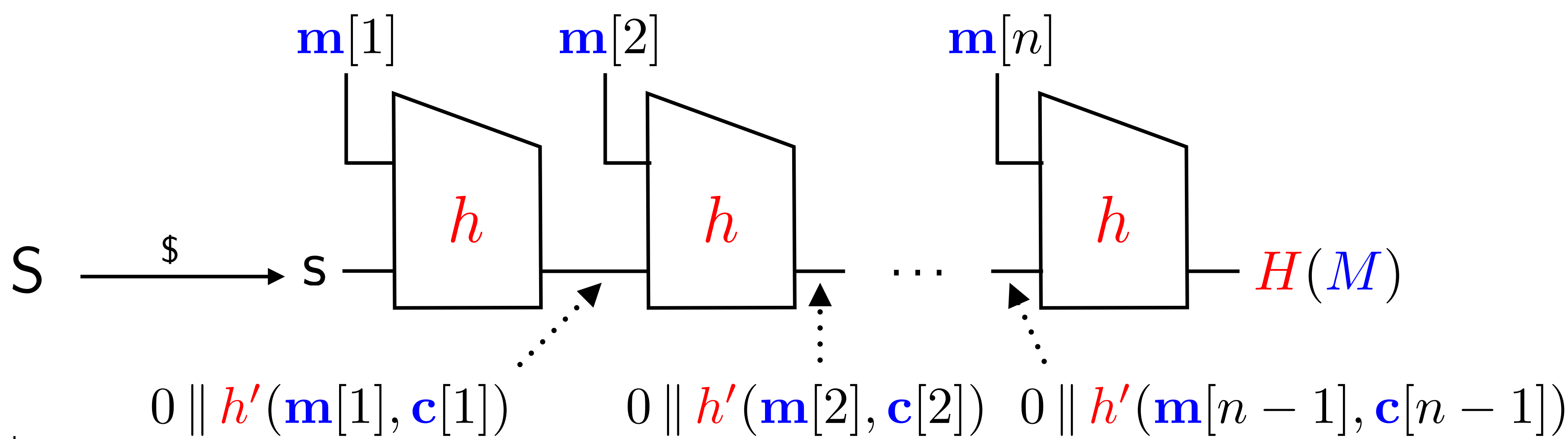

Speeding up MD

Recall: using an injective splitting function could potentially save an extra call to h. This could lead to efficiency gains in the performance of the MD transform.

Theorem

Let Split be an injective splitting function. Given an adversary $\mathcal{A}_H$ we define adversaries $\mathcal{A}_h$ and $\mathcal{B}_h$ such that

$$\mathbf{Adv}_H^{\mathsf{cr}}(\mathcal{A}_H) \leq \mathbf{Adv}_h^{\mathsf{R_{ccr}S}}(\mathcal{A}_h) + \mathbf{Adv}_h^{\mathsf{R_{pre}S}}(\mathcal{B}_h)$$

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[AnSt11] informally state similar result for CR.

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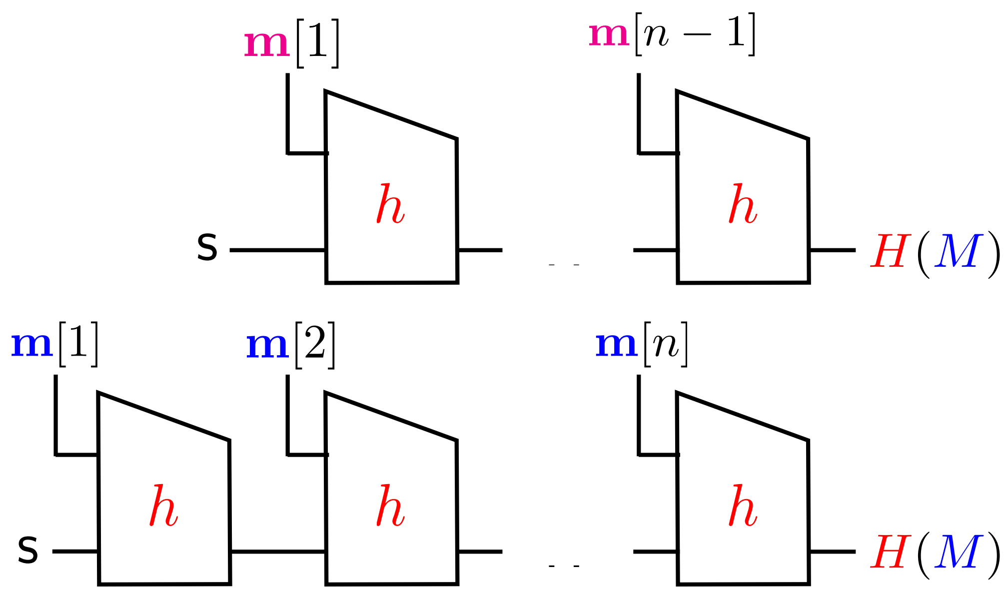

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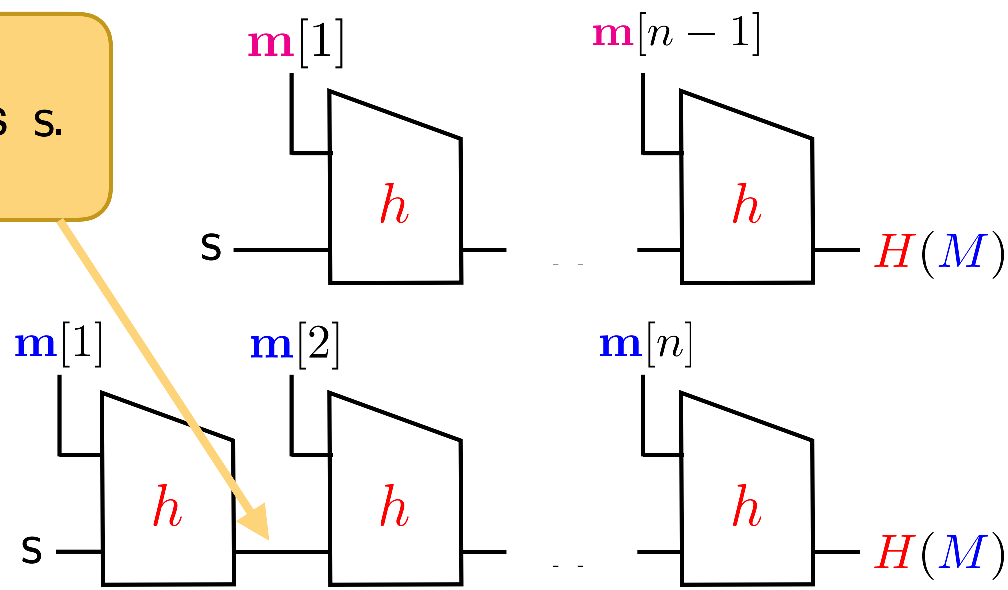

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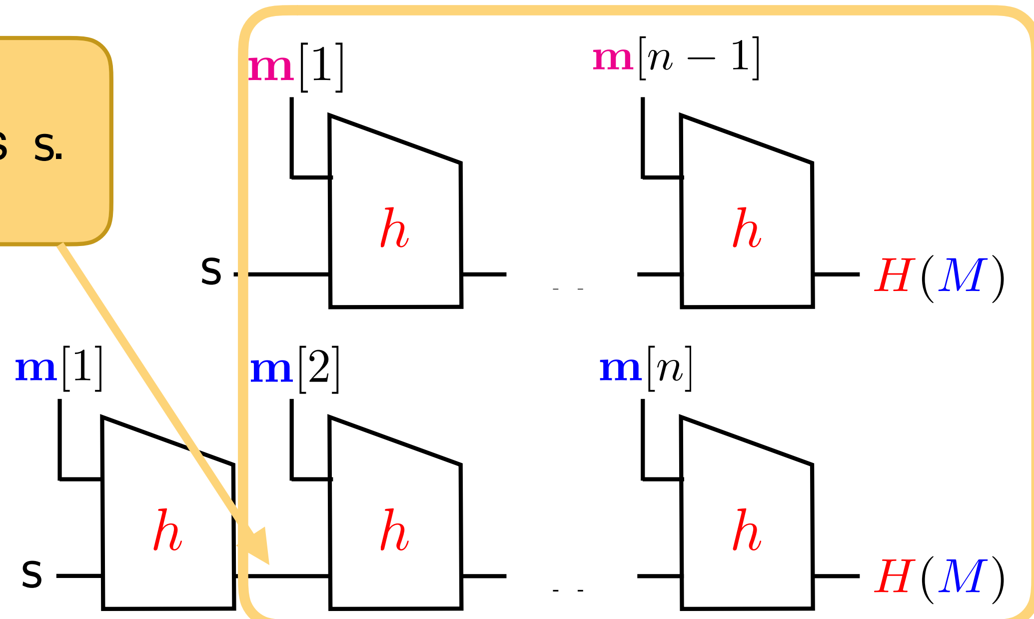
Case 1: This is s.

Case 2: This is a collision in h somewhere here.



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- We looked at memory complexity by explicitly giving reductions. In addition, we gave alternate reduction algorithms that were more memory tight. This allows us to **more easily address memory complexity**.
- We showed how the MD transform can be made **more efficient** by using an **injective splitting function**. In particular, if the splitting function is injective, the compression function is CCR, and it is hard to find a pre-image for s, then the hash function will be CR.